

An analysis and application of the size distribution of waste flakes from the manufacture of bifacial stone tools

David W. Stahle and James E. Dunn

A number of analytical methods have been developed to reconstruct the sequences and techniques involved in the manufacture of stone tools and the resulting waste flakes recovered from prehistoric sites. These methods include the classification of partial and completed tools (e.g., Holmes 1894, 1897; Muto 1971), analysis of assemblage content (e.g., Sheets 1975), analysis of the type and sequence of flake scars on completed tools (Crabtree 1966, 1970), attribute analysis of the resulting waste flakes or debitage (e.g., Montet-White 1963; Shafer 1969; Burton 1980), conjoining of waste flakes to reassemble the original core (e.g., Newcomer 1971; Bunn *et al.* 1980; Cahen and Keeley 1980), and modern experimental replication by expert flintknappers (e.g., Bordes 1961; Crabtree 1966; Flenniken 1978; Callahan 1979). These methods offer convincing evidence that the prehistoric manufacture of many bifacially flaked stone tools typically progressed through several sequential stages of reduction. Stages of biface reduction may provide important technological criteria to differentiate prehistoric sites or activity areas on the basis of lithic remains.

Stages of biface reduction are analytical categories designed to reflect the changing objectives and techniques involved in the continuous reduction of a parent core into a finished tool. The complete reduction sequence, however, need not have been performed at once, or in the same location (e.g., Collins 1975:15-16; Burton 1980:138). For example, large, unfinished bifacial blanks were frequently produced at or near quarry locations and were later completed at other sites (Holmes 1894:14, 1897:26; Montet-White 1968:23-4). Description of the tools and waste flakes recovered from such sites should indicate obvious differences that might imply both technological specialization in the manufacture of projectile points and functional differences between the sites (i.e., possibly an extraction *vs.* habitation site). On the other hand, all stages of biface manufacture may have occurred at the same location in regions rich in high quality raw material (Bordes and de Sonneville-Bordes 1970:66-7), or simply as a result of preference. The important point is that the location of all stages of biface reduction may or may not have changed during the course of manufacture, and this variability in past behavior is precisely what may be identified by a stage analysis of incomplete and finished stone tools or their resulting waste flakes.

The methods to classify or conjoin waste flakes in order to identify the stages of manufacture in stone tools appear theoretically sound, but may be both time consuming and difficult to standardize. To complement these existing procedures, we have developed a rapid, systematic method to (1) help identify unbiased waste flake samples that have resulted from biface

World Archaeology Volume 14 No. 1 *Quantitative methods*

© R.K.P. 1982 0043-8243/82/1401-084 \$1.50/1

reduction, based on the frequency distribution of waste flake size categories, and (2) to estimate the relative proportion of single or multiple stages of biface reduction represented in prehistoric flake assemblages by comparison with experimentally replicated flake standards using a constrained least squares technique. Further experimental studies will be necessary to fully define the potential limitations of the method outlined here, but it should be useful for flake size analysis of bifacial stoneworking industries worldwide, and the general approach may be adapted for the analysis of waste flake remains from other stoneworking industries as well.

Flake size distribution

Casual observation and controlled experimentation (e.g., Newcomer 1971) indicate that the size of waste flakes from the manufacture of bifacial projectile points, knives, or large handaxes will systematically decrease from the initial to final stages of manufacture as the emerging tool is reduced, thinned, and shaped. This underlying regularity suggests that the size distribution of waste flakes may be used to distinguish sequential stages of biface manufacture. The results reported below stem from our empirical studies to test this hypothesis, which formally states that the size range of waste flakes will decrease as biface reduction proceeds and, as a result, this systematic change in flake sizes may be used to identify the stage or stages of reduction present in unknown or prehistoric waste flake samples from biface manufacture by comparison with known flake samples produced through controlled experimental replication. The attempt to determine stage of manufacture based on the size range of waste flakes is somewhat analogous to particle size analysis of sediments to assess the energy regimes of depositional environments (e.g., Folk and Ward 1957).

To begin the experiment we defined four general stages of biface reduction that appear (based on observation of partial and completed tools and debitage) to have been involved in the manufacture of Afton points, a corner-notched dart point or knife of medium size from the Archaic period in the central United States (Bell 1958:6-7). Our experienced flintknapper, Michael Sierzchula, used hammerstones, wood and antler batons, and an antler pressure flaker variously for rough flaking and grinding, bifacial thinning, and delicate final shaping. In the first stage cortex, bedding planes, and other imperfections were removed, and a striking platform and ridge were prepared in order to detach a large flake blank from the primary core of Boone chert. The flake blanks were heat treated in an electric furnace at 350 °C to 400 °C for 96 hours. The preheated blanks were then bifacially flaked to remove any remaining cortex and to begin thinning the cross-section in stage two. Bifacial thinning was continued and completed in Stage 3, and the margin and hafting area were bifacially shaped to finish the replica in Stage 4 (Raab *et al.* 1979; Stahle and Dunn n.d.).

All waste flakes produced during twelve separate replications were separated into these four general stages, including all broken and whole flakes, chips, spalls, and shatter. The replicated flake samples were identified by replication and stage of reduction and each was then sieved through 10 graduated screen sizes and all flakes in each size category were counted. Since flake screening was standardized by passing all flakes that would fit at any angle through a given sieve (graduating flakes consistently by their minimum diameter), the manipulation of some flakes in each sieve was necessary. Nevertheless, this procedure is quick, replicable, and systematic. Flakes smaller than 1/8 inch were not counted, resulting in ten size categories and a

left-censored size distribution. The nominal size of the hardware cloth screens used in this experiment ranged from 1/8 to 2 inches (with the maximum size of the square openings actually measuring diagonally at: 4.5, 9.0, 13.5, 18.0, 22.5, 27.0, 31.5, 36.0, 53.9, and 71.8 mm), and a constant interval of 1/8 inch was maintained between all but the two largest size categories. The two largest size categories have had a minimal effect, however, since only eleven flakes were retained in the two largest screens. (The complete details of the experiment, analysis, implications, and potential limitations of the method outlined here are discussed by Stahle and Dunn n.d.).

All broken and whole flakes were included in this analysis for three reasons: (1) Biface manufacture is a reductive process (Collins 1975:16) so that the maximum size of broken as well as whole flakes is constrained by the decreasing size of the biface as manufacturing proceeds; (2) Flakes appear to break in a random fashion during all stages of reduction; and (3) Including all flakes eliminates the time consuming necessity of sorting whole flakes.

In the constrained least squares analysis discussed below, two replicated flake samples from each stage of reduction were randomly selected to serve as blind test cases, and the flake frequency data for the remaining replicated samples were pooled by size category to define the known standards for each stage of reduction. Additional test cases representing mixtures of two, three, or four reduction stages were simulated by combining data from the single stage test cases by size category.

As might be expected, a simple plot of flake frequency by size and category is completely dominated by the many small flakes produced during all stages of reduction. This trend in flake

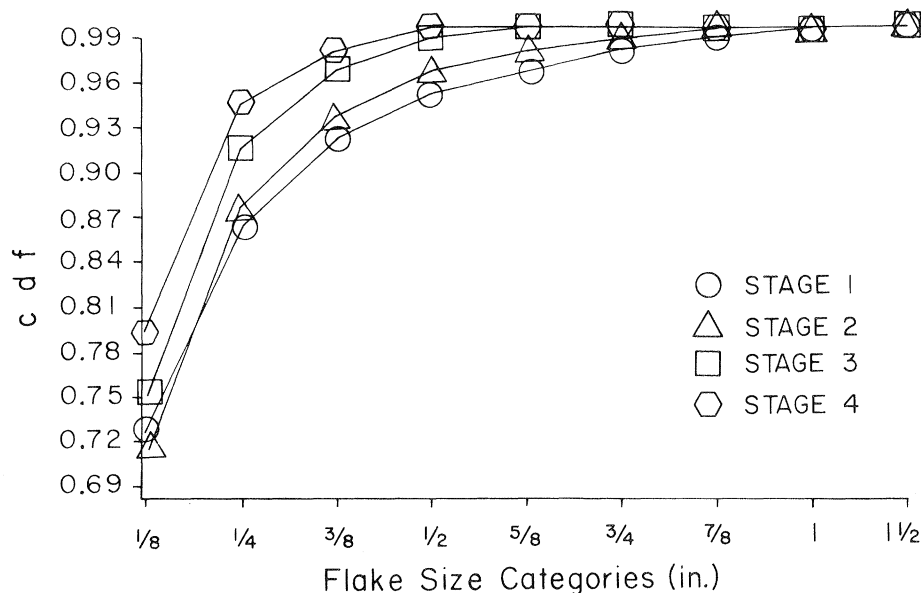


Figure 1 Cumulative distribution functions of flake frequency by size category. Note the increasing proportion of smaller flake size categories from Stage 1 to Stage 4. Each reduction stage represents the average of several replications combined by size category to define the four known stage standards used in the constrained least squares analysis.

size is consistent among all replications at each stage of reduction and can best be expressed by a plot of the cumulative distribution function (cdf) of flake frequency by size category. Fig. 1 depicts average cumulative distribution functions for each stage of reduction. The smallest flake size categories dominate the distributions, but become increasingly important from Stage 1 to Stage 4.

Given this trend in flake size, we attempted to find a family of theoretical distributions which would most accurately model the flake size distribution for all individual or pooled replications at all stages of reduction. Our motives were based on three considerations: (1) Since the respective samples varied considerably in the total frequency of flakes, fitting a theoretical probability distribution was an effective means of scaling in order to remove the effects of sample size; (2) For the purpose of characterizing each of the reduction stages, it seemed essential to do so by making an inference about the background populations from which the current samples were drawn. Fitting theoretical distributions to empirical distributions served effectively as a smoothing operation in this application in order to remove the effects of sampling variation from the final estimates of flake size distributions; and (3) An accurate theoretical model of the size distribution of waste flakes could potentially be used as a screening device to help assure that a particular prehistoric flake sample does, in fact, represent the remains of biface manufacture and has not been biased by the waste of other technologies, prehistoric selection of certain flake sizes, secondary deposition, or modern relic hunting.

Our search for an appropriate theoretical model was restricted to the following three families of distributions, which are described below by their density functions $f(d)$ and cdf $F(d) = P[D \leq d]$ as a function of flake size diameter d and parameters a , b , and c :

Exponential $f(d) = a e^{-ad}$

$$F(d) = 1 - e^{-ad}$$

where $d > 0$ and $a > 0$.

Weibull $f(d) = \frac{c}{a} \left(\frac{d-b}{a}\right)^{c-1} \exp\left\{-\left(\frac{d-b}{a}\right)^c\right\}$

$$F(d) = 1 - \exp\left\{-\left(\frac{d-b}{a}\right)^c\right\}$$

where $d > b$ and $a > 0$.

Extreme value $f(d) = \frac{1}{a} \exp\left\{-\left(\frac{d-b}{a}\right)\right\} \exp\left\{-\exp\left[-\left(\frac{d-b}{a}\right)\right]\right\}$

$$F(d) = \exp\left\{-\exp\left[-\left(\frac{d-b}{a}\right)\right]\right\}$$

where $d > b$ and $a > 0$.

Both the Weibull and the extreme value distributions historically have been used in applications involving the stress testing and fracturing of materials (cf. Gumbel 1958). The exponential is a special case of the Weibull with $c = 1$. Our chief justification for proposing the foregoing distributions is that they all lend themselves to the empirical requirement of being long tailed, positively skewed distributions.

The problem of fitting any of these distributions was somewhat confounded in two respects by the sampling procedure. First, the flakes were naturally grouped as a result of the graduated nature of the screening process. Hence, the usual estimation procedures (i.e., maximum likelihood or the method of moments) are not applicable. Second, the frequency of flakes passing through the smallest sieve size was not measured, so that the observed cdf was automatically censored on the left with respect to all flakes capable of passing an $\frac{1}{8}$ -inch sieve. The exponential distribution is insensitive to left-censoring since the resulting conditional density is

$$f(d|D > \frac{1}{8}) = \frac{a e^{-ad}}{P[D > \frac{1}{8}]} = \frac{a e^{-ad}}{e^{-a/8}} = a e^{-a(d - \frac{1}{8})}$$

$$\text{with cdf} \quad F(d|D > \frac{1}{8}) = 1 - e^{-a(d - \frac{1}{8})} \quad (1)$$

for $d > \frac{1}{8}$, i.e., still exponential. The exponential distribution is unique in this respect. However, it is convenient to postulate that the censored distribution may be approximately Weibull with $b = \frac{1}{8}$ and resulting cdf

$$F(d|b = \frac{1}{8}) = 1 - \exp \left[- \left(\frac{d - \frac{1}{8}}{a} \right)^c \right] \quad (2)$$

for $d > \frac{1}{8}$, or that the censored distribution approximately follows the extreme value probability law on $d - \frac{1}{8}$, corresponding to the cdf

$$F(d - \frac{1}{8}) = \exp \left\{ - \exp \left[- \left(\frac{d - \frac{1}{8} - b}{a} \right) \right] \right\} \quad (3)$$

For the purpose of model selection, it is convenient to note that equations (1) through (3) are intrinsically linear. Specifically, the linearized forms may be obtained as

$$\text{Exponential} \quad - \ln(1 - F) = a(d - \frac{1}{8}) \quad (4)$$

$$\text{Weibull} \quad \ln[-\ln(1 - F)] = c \ln(d - \frac{1}{8}) - c \ln a \quad (5)$$

$$\text{Extreme value} \quad - \ln[-\ln F] = 1/a(d - \frac{1}{8}) - b/a \quad (6)$$

In order to distinguish between these models, let c_i represent the fraction of flakes which passed through sieve diameter d_i ($i = 2, 3, \dots, K$), aside from that lost through the smallest screen. Thus $0 \leq c_2 \leq c_3 \leq \dots \leq c_K \leq 1$ represents an empirical estimate of the theoretical censored cdf $F(\cdot)$ corresponding to K mesh sizes $d_1 < d_2 < \dots < d_K$. If the distribution of flakes is exponential, then from equation (4), a plot of $-\ln(1 - c_i)$ vs. $(d_i - \frac{1}{8})$ should be essentially linear with zero intercept. Similarly, a linear relationship between $\ln[-\ln(1 - c_i)]$ and $\ln(d_i - \frac{1}{8})$ with possibly a nonzero intercept would suggest a Weibull distribution, while a linear relationship between $-\ln[-\ln c_i]$ and $(d_i - \frac{1}{8})$ with a nonzero intercept would suggest the extreme value distribution.

When each of these plots was made for all individual and combined replications and appropriate linear regression lines fitted, the Weibull gave the closest fit of observed and estimated values (cf. fig. 2) and the highest correlation for the vast majority of cases (Stahle and Dunn n.d.). The coefficient of determination (r^2) for the flake size data combined over all replications and stages of reduction to define the sample population of biface reduction flakes was 0.9950 with the Weibull, 0.9326 with the extreme value, and 0.9717 with the exponential distribution.

This analysis suggests that the Weibull is an accurate model of the size distribution for flake

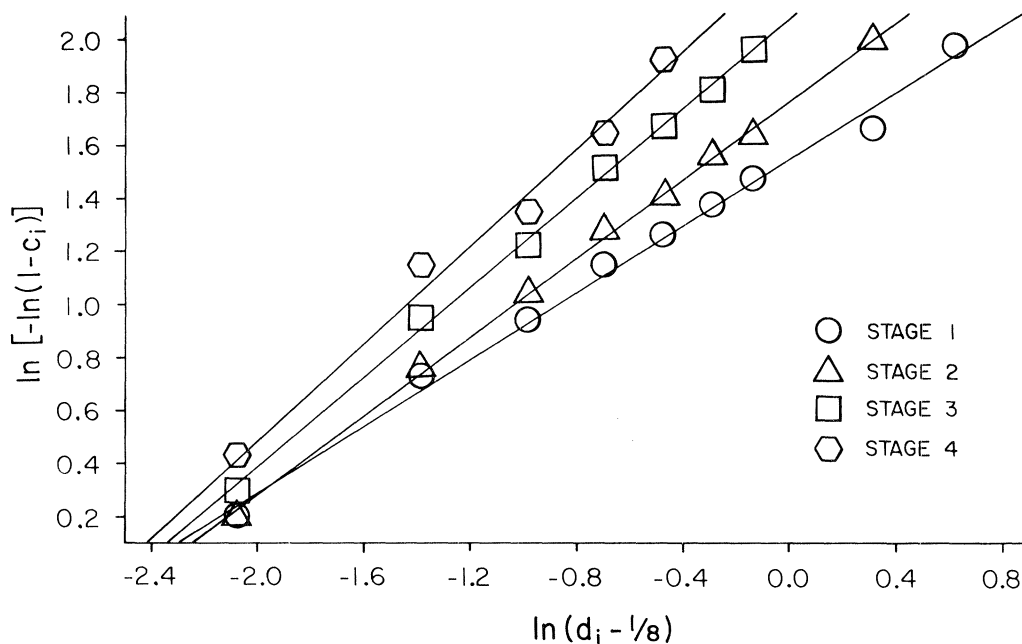


Figure 2 This plot illustrates the linear relationships which are obtained when the Weibull transformation ($\ln[-\ln(1 - c_i)]$) is applied to the cdf of flake size categories ($\ln(d_i - \frac{1}{8})$) for each stage of reduction. The apparent linear relationship offers strong evidence that the long tailed, positively skewed Weibull distribution is an accurate model of the waste flake size distribution from biface reduction

frequency from biface reduction. Although the exponential and extreme value distributions gave slightly higher correlations for some individual replications, the Weibull distribution consistently gave the best correspondence between observed and estimated curves. The Weibull fit the raw data best when the replications were pooled because the cdfs were better defined with the larger sample sizes.

These results tend to substantiate our contention that the Weibull distribution may be used as a screening device to help assure that a particular prehistoric flake sample represents the unbiased remains of biface reduction. If the Weibull distribution is presumed to be an accurate model of the frequency distribution of all flake sizes from biface reduction, random samples drawn from a population of biface reduction flakes should also reflect the proportion of flake sizes defined by the Weibull distribution. Thus, a properly sampled and unbiased prehistoric assemblage of waste flakes from biface reduction should conform to the Weibull distribution, while biface reduction flakes which have undergone extensive prehistoric selection, secondary deposition, modern relic hunting, or were deposited with waste from different stoneworking technologies *may* have very different flake size distributions. In actual practice, the simplest approach to assess bias would be to plot the Weibull transformation ($\ln[-\ln(1 - c_i)]$) against the cdf of flake size categories ($\ln(d_i - \frac{1}{8})$) and inspect the result for linearity (e.g., fig. 2). Further experimentation will be necessary, however, to define the size distribution of waste flakes for other major stoneworking technologies before it will be possible to link a sample of waste flakes following the Weibull distribution solely to biface reduction. In the many regions and time periods where biface technologies predominate, however, this should not be a serious problem.

Constrained least squares analysis

Since the waste flakes from bifacial reduction on archaeological sites may often represent a mixing of two or more stages of reduction, a method of analysis is needed that will estimate the relative contribution that each component stage makes to the distribution of an unknown or prehistoric flake assemblage. If the four known stages of reduction are defined by curves representing the cdf of waste flakes in the ten graduated screen sizes, then an unknown flake collection may be assigned to one or more of these stages by plotting its cdf of flake sizes and asking what linear combination of the standard cdf curves best reproduces that of the unknown (cf. fig. 3). Formally, if c_{ij} represents the cdf of the known standard for the i th screen size and the j th stage of reduction ($i = 1, 2, \dots, K; j = 1, 2, 3, 4$) and u_i represents the cdf of the unknown for the i th screen size, then we postulate that

$$u_i = P_1 c_{i1} + P_2 c_{i2} + P_3 c_{i3} + P_4 c_{i4} + \epsilon_i \quad (7)$$

($i = 1, 2, \dots, K$), where ϵ_i represents a random deviation from the expected linear combination of the standards. The constants P_1, P_2, P_3 , and P_4 represent the unknown proportions

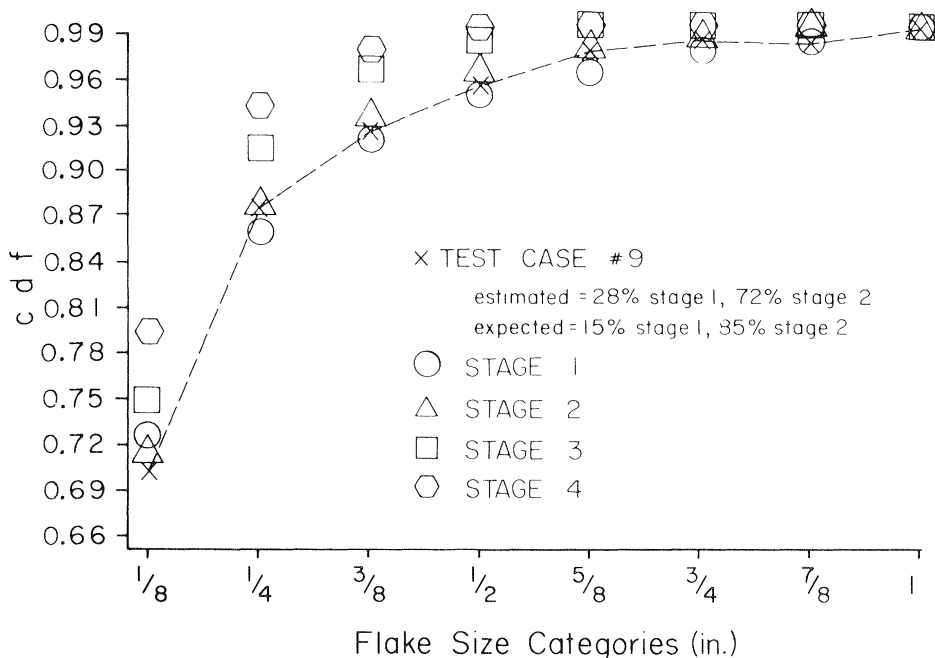


Figure 3 The sequential stages involved in the manufacture of a bifacial tool may be defined by the cdf of waste flakes produced under controlled experimental replication (e.g., Stages 1 through 4), and the stage or stages of reduction present in an unknown or prehistoric flake sample may be determined by comparison with the replicated standards through constrained least squares analysis. In this example, the blind test case (no. 9 in Table 1) was manufactured during the replication experiment to serve as an unknown, but was not included with the samples defining the four stage standards. Constrained least squares analysis (GLS solution) between test case no. 9 and the four stage standards gave a reasonable approximation (estimated) of the expected relative proportions of Stages 1 and 2 in this unknown test case. Table 1 presents the complete results of the blind test cases

and must be estimated from the data. (The number of constants may be adjusted to accommodate the number of stages specified by the particular model of biface manufacture proposed.) Pasternak (1962), in an analogous problem of estimating the amounts of known radionuclides present in a mixture, proposed a weighted least squares solution based on Poisson assumptions for the counts in the individual categories. Dunn (1969), in the context of estimating yields of various activated states of helium electrons with this model, pointed out that the physical constraints $\sum_i P_i = 1.00$ with $P_i \geq 0$ must be satisfied in order to account for 100 per cent of the unknown and to guarantee nonnegative estimates of the components. He proposed that a quadratic programming algorithm be used to obtain the constrained, *ordinary* least squares (OLS) estimates. This approach was proposed simultaneously by Mantel (1969). Using the modern SAS-based version of the Theil-Van de Panne quadratic programming algorithm supplied by Sall (1979, personal communication) we obtained a comparison of two approaches to constrained estimation of $P_1 - P_4$ in equation (7). In the first case, the constraints were imposed on the OLS solution, i.e., without any sort of weighting. For a more detailed analysis, the (left-censored) frequency distribution of flakes over the respective screen sizes was assumed to follow a multinomial sampling distribution. Under these assumptions, the covariance matrix for the unknown cdf $0 < u_2 \leq u_3 \leq \dots \leq u_K < 1$ is readily obtained (e.g., Rao 1952), so that the second approach consists of imposing the required physical constraints on the *generalized* least squares (GLS) solution (cf. Searle 1971). This solution has considerable theoretical appeal since it more properly accounts for the obvious intercorrelations among successive values of the empirical cdf. In this comparative analysis the GLS solution consistently gave the best estimates for known samples, presumably because the smaller screen size values are subject to less sampling variation and are thus preferentially weighted in the GLS model (Stahle and Dunn n.d.). The constrained least squares (LS) results discussed below are based on the GLS model. (The listing of an SAS program to perform this analysis of flake size data is available in Stahle and Dunn n.d.)

To test the usefulness of constrained LS analysis for assigning unknown flake collections to stages of biface reduction, we randomly selected two replications for each stage of reduction (eight in all) to serve as single stage test cases. Data from seven of these single stage test cases were then combined by screen size into fourteen additional test cases so that we could examine the ability of the LS analysis to identify unknown flake samples consisting of mixtures of two, three, or four reduction stages (Stahle and Dunn n.d.). Values of the cdf for the known standards were taken from fitted Weibull distributions in each case. Using these smoothed values for the known standards stabilized the curves and allowed estimation of missing values for the larger screen sizes in Stages 2, 3, and 4. Similarly, the Weibull distribution was fitted to the test cases (unknowns) as a preliminary device to obtain a smoothed estimate of the covariance matrix for GLS. For actual estimation of $P_1 - P_4$, however, the test cases were not smoothed, but were simply plotted and fitted as the empirical cdf against the four known stage standards. The accuracy of the fit between the known stage estimation and the cdf of the unknown sample is apparent in the linear plots of observed and fitted curves and is roughly measured by the multiple correlation coefficient (r^2).

Table 1 lists the results for GLS estimation of the single and mixed stage test cases. An example of the cdf plot of a mixed stage test case against the four stage standards is shown in fig. 3.

Examination of Table 1 indicates that the constrained GLS solution correctly assigned most of the single stage test cases to their appropriate stage of reduction. The contributions of the

Table 1 Results of the constrained least squares analysis (GLS solution) for 22 blind test cases against the four sequential stages of biface reduction defined with waste flake size data from the controlled replication experiment. The first eight test cases are composed entirely of flakes from a single stage of reduction; test cases 9 through 22 are mixtures of various stages (as listed in the left column) and were obtained by combining the flake frequency data by size category for two or more single stages. The results are expressed as percentage assignments to one or more of the four stage standards (est column), and the expected percentages are listed in italics (exp column). The correct results should fall within the boxed areas. Distance values (*d*) measure accuracy in reproducing the expected stage percentages for each test case (on a scale of 0 to 1.414, see text), and the median *d* value and standard deviation are computed for the test cases in the single and mixed stage groups.

		Single stage test cases								Distance value (<i>d</i>)
Test case number		Stage 1		Stage 2		Stage 3		Stage 4		
		est	exp	est	exp	est	exp	est	exp	
Stage 1	1	0.71	<i>1.0</i>					0.29		0.41
	2	0.74	<i>1.0</i>					0.26		0.39
Stage 2	3	0.58		0.42	<i>1.0</i>					0.82
	4	0.05		0.95	<i>1.0</i>					0.07
Stage 3	5	0.22		0.23		0.55	<i>1.0</i>			0.55
	6			0.17		0.83	<i>1.0</i>			0.24
Stage 4	7	0.03						0.97	<i>1.0</i>	0.04
	8					0.13		0.87	<i>1.0</i>	0.18
Median <i>d</i> = 0.315										
sd = 0.262										
		Mixed stage test cases								Distance value (<i>d</i>)
Test case number		Stage 1		Stage 2		Stage 3		Stage 4		
Stages 1,2	9	0.28	<i>0.15</i>	0.72	<i>0.85</i>					0.19
	10	0.05	<i>0.03</i>	0.95	<i>0.97</i>					0.02
Stages 1,2,3	11	0.22	<i>0.10</i>	0.67	<i>0.55</i>	0.11	<i>0.35</i>			0.29
	12	0.18	<i>0.11</i>	0.77	<i>0.67</i>	0.05	<i>0.22</i>			0.21
Stages 1,2,3,4	13	0.27	<i>0.09</i>	0.44	<i>0.50</i>	0.29	<i>0.32</i>		<i>0.09</i>	0.22
	14	0.02	<i>0.02</i>	0.50	<i>0.51</i>	0.48	<i>0.17</i>		<i>0.30</i>	0.43
Stages 2,3	15			0.98	<i>0.61</i>	0.02	<i>0.39</i>			0.52
	16			1.00	<i>0.75</i>		<i>0.25</i>			0.35
Stages 2,3,4	17	0.07		0.71	<i>0.55</i>	0.22	<i>0.35</i>		<i>0.10</i>	0.24
	18	0.02		0.50	<i>0.52</i>	0.48	<i>0.17</i>		<i>0.31</i>	0.43
Stages 3,4	19	0.24				0.64	<i>0.77</i>	0.12	<i>0.23</i>	0.29
	20					0.63	<i>0.36</i>	0.37	<i>0.64</i>	0.38
	21	0.11				0.70	<i>0.52</i>	0.19	<i>0.48</i>	0.36
	22					0.88	<i>0.64</i>	0.12	<i>0.36</i>	0.33
Median <i>d</i> = 0.310										
sd = 0.125										

various stages were also identified with reasonable accuracy for the mixed test cases. While the major stage composition of the test cases was generally well approximated, over 25 per cent of the cdf were assigned to the wrong stage of reduction for some single and mixed test cases.

Due to this problem and to assist the visual evaluation of the results in Table 1, 'distance' values were computed between the estimated and expected compositions for single and mixed stage test cases to assess the accuracy of the LS analysis in reproducing the known composition of test cases. Distance values are computed from the results in Table 1 as follows:

$$d = \sqrt{(P_{1\text{est}} - P_{1\text{exp}})^2 + (P_{2\text{est}} - P_{2\text{exp}})^2 + (P_{3\text{est}} - P_{3\text{exp}})^2 + (P_{4\text{est}} - P_{4\text{exp}})^2}$$

where $P_{i\text{est}}$ = the estimated percentage stage assignment of the test case, and $P_{i\text{exp}}$ = the expected (known) percentage stage composition of the test case for $i = 1, 2, 3, 4$.

Geometrically, d is the Euclidean distance between two points on the face of an r -dimensional simplex, where one point represents the true composition and the second represents the estimate. The distance value equals zero when estimated and expected values are identical and increases as the difference between estimated and expected proportions increases. The maximum distance value possible for $r \geq 2$ is $\sqrt{2}$ or approximately 1.414, which represents the worst possible solution for the assignment of a test case of known composition.

The distance values between the estimated and expected results of the LS analysis of single and mixed stage test cases are also listed in Table 1. The median d value for the mixed stage test cases indicates that half of the test cases are closer than 0.310 to the true proportions, suggesting good agreement between the estimated and expected flake size proportions.

The median d value and standard deviation for the single stage test cases were higher than similar results for mixed stage samples (Table 1). This was expected because boundary or single stage samples are theoretically more difficult to assign with precision than mixed samples. In terms of the geometry of a simplex, a true value at the vertex of the simplex, corresponding to a single stage sample, can be further away from the estimate than would be possible for a mixed stage sample where the true value lies in the interior of the simplex.

Although the results in Table 1 indicate considerable success in estimating the relative compositions of single and mixed stage test cases with constrained least squares analysis, the expected stage percentages are not exactly reproduced and a strict interpretation of these percentages could be misleading (e.g., test cases nos. 1 and 15, Table 1). If a conservative approach is taken, however, and the interpretation is limited to the identification of initial-middle-final reduction or some combination of the three, the constrained LS estimates would be correct for over 75 per cent of the single and mixed stage test cases. We could expect a similar level of accuracy when we compare unbiased waste flakes of biface reduction from prehistoric sites with replicated flake collections from known stages of reduction. In retrospect, a simpler model of biface reduction defining just three manufacturing stages could be used, which would probably improve the accuracy of test results and should be adequate for most archaeological applications.

Discussion

This analysis indicates that the Weibull probability distribution is an accurate model for the size distribution of waste flakes from biface reduction. This analysis also demonstrates that the four reduction stages defined for this experiment are distinctive and may be used with reasonable accuracy in a constrained least squares analysis to assign unknown flake samples produced by

similar methods to their appropriate stage of reduction. When multiple manufacturing stages are mixed in a single sample, the LS analysis provides an objective method to recognize mixing and to determine the approximate relative abundance of the various stages represented. This procedure is successful because the bifacial reduction of stone tools produces a range of flake sizes that become progressively smaller as manufacturing proceeds. At progressive points or stages during manufacture, the decreasing size of the biface is reflected by a change in the frequency distribution of flake sizes. The cdf over flake size categories, therefore, provides an empirical basis to identify stages in the manufacture of bifacially flaked stone tools.

The method described above is quick, replicable, systematic, fully independent of typological approaches, and may be applied to representative collections with as few as 50 to 100 flakes or to large flake samples from major excavations or surface surveys. The Weibull distribution may be used as a screening or testing device to help assure that particular flake samples are the unbiased result of biface reduction, since analysis has shown the Weibull to be an accurate model of the size distribution of waste flakes from biface reduction.

Measurements of flake shape (primarily the length and width of whole flakes) have been previously used with some success to characterize prehistoric assemblages (Wilmsen 1970), and to portray chronological changes in stoneworking technology (Pitts and Jacobi 1979). A recent study examined ten meticulously measured whole flake variables to characterize the reduction sequence of bifacial hand axe manufacture, and found that one variable, mass or weight, accounted for 76 per cent of the discrimination between flake categories from initial to final stages of manufacture (Burton 1980). Since flake weight is a direct expression of flake size, Burton's analysis tends to substantiate our position that simple flake size is adequate to characterize sequential stages of biface reduction. We examined flake weight and found that the total weight of flakes by size category was also accurately modelled by the Weibull probability distribution. Similar, although somewhat less accurate results, were obtained when the constrained LS analysis was based on flake weight rather than frequency. For a discussion of the theoretical problems involved in using frequency *vs.* weight data as a basis for constrained LS analysis, see Stahle and Dunn (n.d.).

Many factors will have to be considered before we can confidently apply the size distribution of waste flakes from biface reduction to typical archaeological problems. These potential problems may include the use of different manufacturing methods to achieve the same product, the combined deposition of waste flakes from different stoneworking technologies, inaccurate replication of the particular stoneworking tradition of interest, the effect of lithic raw materials, prehistoric selection of certain flake size categories, and modern relic hunting.

As complex as these potential problems may appear, they are not insurmountable since their potential effects on the flake size distribution can be readily examined through controlled replication experiments. The necessary experimental work to replicate the biface tradition of interest and to examine potential problems is certainly justified, however, by the speed and objectivity of the flake size analysis outlined here and the prevalence of bifacial stoneworking technologies in world prehistory. Ultimately, the diagnostic value of flake size data can only be as accurate as the archaeologist's model of a prehistoric stoneworking technology. There will be no substitute for careful archaeological description and interpretation of lithic industries, and expert replication of the probable methods involved in the manufacture of stone tools.

Acknowledgments

This research was sponsored by the Arkansas Archeological Survey and the University of Arkansas. The experiment was initiated by L. Mark Raab, and the replication was performed by Michael Sierzchula. Howard Houston and Susan Fennell assisted the analysis, and Robert Cande, Steven Imhoff, Raymond Medlock, Fredrick Limp, Deborah Sabo, George Sabo and Marvin Kay offered valuable advice and criticism that substantially improved the manuscript. The authors, however, are solely responsible for any errors of fact or interpretation.

5.xii.1981

Arkansas Archeological Survey
University of Arkansas

References

- Bell, R. E. 1958. Guide to the identification of certain American Indian projectile points. *Oklahoma Anthropological Society Bulletin* (Oklahoma City). 1.
- Bordes, F. 1961. *Typologie du Paléolithique ancien et moyen*. Bordeaux: Delmas.
- Bordes, F. and de Sonneville-Bordes, D. 1970. The significance of variability in Palaeolithic assemblages. *World Archaeology*. 3(1):61-73.
- Bunn, H., Harris, J. W. K., Isaac, G., Kaufulu, Z., Kroll, E., Shick, K., Toth, N. and Behrensmeier, A. K. 1980. FxJj50: an early Pleistocene site in northern Kenya. *World Archaeology*. 12(2): 109-36.
- Burton, J. 1980. Making sense of waste flakes: new methods for investigating the technology and economics behind chipped stone assemblages. *Journal of Archaeological Science*. 7:131-48.
- Cahen, D. and Keeley, L. H. 1980. Not less than two, not more than three. *World Archaeology*. 12(2):166-80.
- Callahan, E. 1979. The basics of biface knapping in the eastern fluted point tradition, a manual for flintknappers and lithic analysts. *Archaeology of Eastern North America*. 7(1):1-180.
- Collins, M. B. 1975. Lithic technology as a means of processual inference. In *Lithic Technology* (ed. E. H. Swanson, Jr.). The Hague: Mouton, pp. 15-34.
- Crabtree, D. E. 1966. A stoneworker's approach to analyzing and replicating the Lindenmeier Folsom. *Tebiwa*. 9(1):3-39.
- Crabtree, D. E. 1970. Flaking stone with wooden implements. *Science*. 169:146-53.
- Dunn, J. E. 1969. The Theil-Van de Panne algorithm in constrained least squares estimation. *Technometrics*. 11:214.
- Flenniken, J. J. 1978. Reevaluation of the Lindenmeier Folsom: a replication experiment in lithic technology. *American Antiquity*. 43(3):473-80.
- Folk, R. L., and Ward, W. C. 1957. Brazos River bar, a study in the significance of grain size parameters. *Journal of Sedimentary Petrology*. 27:3-27.
- Gumbel, E. J. 1958. *Statistics of Extremes*. Columbia University Press.
- Holmes, W. H. 1894. An ancient quarry in Indian territory. *Smithsonian Institution, Bureau of Ethnology Bulletin*. 21:1-19.

- Holmes, W. H. 1897. Stone implements of the Potomac-Chesapeake tidewater province. *Smithsonian Institution, Bureau of Ethnology, Fifteenth Annual Report 1893-1894*, pp. 3-149.
- Mantel, H. 1969. Restricted least squares regression and convex quadratic programming. *Technometrics*. 11:763-4.
- Montet-White, A. 1963. Analytic description of the chipped stone industry from Snyders site, Calhoun County, Illinois. *Anthropological Papers*, no. 19. Ann Arbor: University of Michigan, Museum of Anthropology.
- Montet-White, A. 1968. The lithic industries of the Illinois Valley in the Early and Middle Woodland period. *Anthropological Papers*, no. 35. Ann Arbor: University of Michigan, Museum of Anthropology.
- Muto, G. R. 1971. A stage analysis of the manufacture of chipped stone implements. In *Great Basin Anthropological Conference 1970: Selected Papers* (ed. C. M. Aikens). University of Oregon Anthropological Papers, 1, pp. 109-18.
- Newcomer, M. H. 1971. Some quantitative experimentation in handaxe manufacture. *World Archaeology*. 3:85-93.
- Pasternak, B. S. 1962. Linear estimation and analysis of gamma ray pulseheight spectra. *Technometrics*. 4:565-71.
- Pitts, M. W. and Jacobi, R. M. 1979. Some aspects of change in flaked stone industries of the Mesolithic and Neolithic in Southern Britain. *Journal of Archaeological Science*. 6:163-77.
- Raab, L. M., Cande, R. F. and Stahle, D. W. 1979. Debitage graphs and Archaic settlement patterns in the Arkansas Ozarks. *Midcontinental Journal of Archaeology*. 4(2):167-82.
- Rao, C. R. 1952. *Advanced Statistical Methods in Biometric Research*. New York: Wiley.
- Sall, J. 1979. Personal communication. SAS Institute, Raleigh, North Carolina.
- Searle, S. R. 1971. *Linear models*. New York: Wiley.
- Shafer, H. J. 1969. Archeological investigations in the Robert E. Lee Reservoir basin, west central Texas. *Papers of the Texas Archeological Salvage Project*, no. 17.
- Sheets, P. D. 1975. Behavioral analysis and the structure of a prehistoric industry. *Current Anthropology*. 16(3):369-91.
- Stahle, D. W., and Dunn, J. E. n.d. An experimental analysis of the size distribution of waste flakes from biface reduction. *Technical Series* no. 2. Fayetteville: Arkansas Archeological Survey (in press).
- Wilmsen, E. N. 1970. Lithic analysis and cultural inference. *Anthropological Papers*, University of Arizona Press, no. 16.

Abstract

Stahle, David W. and Dunn, James E.

An analysis and application of the size distribution of waste flakes from the manufacture of bifacial stone tools

A replication experiment tested the hypothesis that the size range of waste flakes from biface manufacture decreases from initial to final reduction stages and may be used to estimate biface

reduction stages in prehistoric flake samples by comparison with replicated flake data. All waste flakes from the replication of twelve projectile points were separated into four experimentally defined stages, and were then sieved into ten size categories. The size distribution was based on the cumulative distribution function of flake frequency by size category, and was accurately modeled by the Weibull distribution. Constrained least squares analysis successfully assigned most single and multiple stage test samples to their correct stage of reduction. Based on careful replication, this method should allow the rapid, systematic estimation of biface reduction stages in unbiased prehistoric flake samples. The Weibull distribution may potentially serve as a screening device to help assure that a particular prehistoric flake sample represents the unbiased remains of biface manufacture.